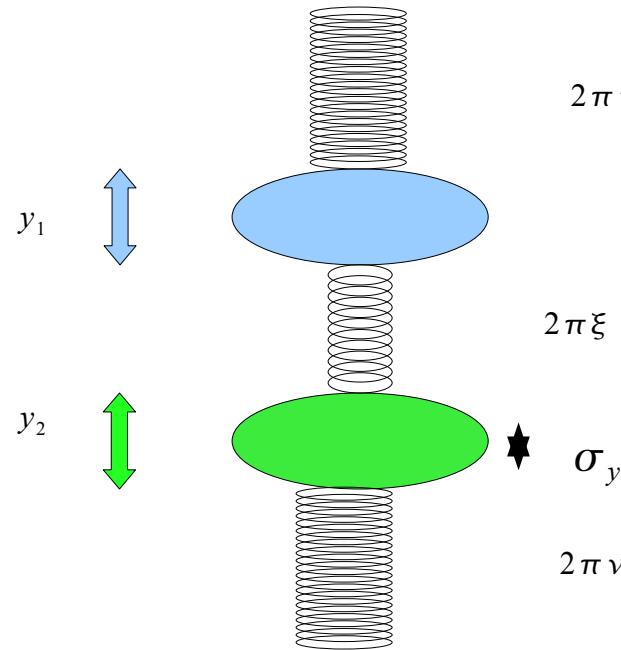


Beam-beam model



ν is the base tune

$\xi = \pm \frac{N r_0 \beta_T}{2\pi \gamma_L \sigma_y (\sigma_x + \sigma_y)}$ is the beam-beam coupling parameter
– for opposite charged beams

$$y_1'' + (2\pi\nu)^2 y_1 = (2\pi\xi)^2 (y_2 - y_1)$$

$$y_2'' + (2\pi\nu)^2 y_2 = (2\pi\xi)^2 (y_1 - y_2)$$

two solutions

$$(y_1 + y_2) \propto e^{\pm 2\pi i \nu} (\sigma \text{ mode})$$

$$(y_1 - y_2) \propto e^{\pm 2\pi i (\nu - \xi)} \quad (\xi \ll \nu) \quad (\pi \text{ mode})$$

Beam-beam model with maps

$$\begin{pmatrix} y_1 \\ y'_1 \\ y_2 \\ y'_2 \end{pmatrix}_{BB} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{2\pi\xi}{\beta} & 1 & \frac{2\pi\xi}{\beta} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{2\pi\xi}{\beta} & 0 & \frac{-2\pi\xi}{\beta} & 1 \end{pmatrix}}_{\text{beam-beam kick}} \underbrace{\begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu & 0 & 0 \\ -\gamma \sin\mu & \cos\mu - \alpha \sin\mu & 0 & 0 \\ 0 & 0 & \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ 0 & 0 & -\gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix}}_{\text{uncoupled one-turn map}} \begin{pmatrix} y_1 \\ y'_1 \\ y_2 \\ y'_2 \end{pmatrix}$$

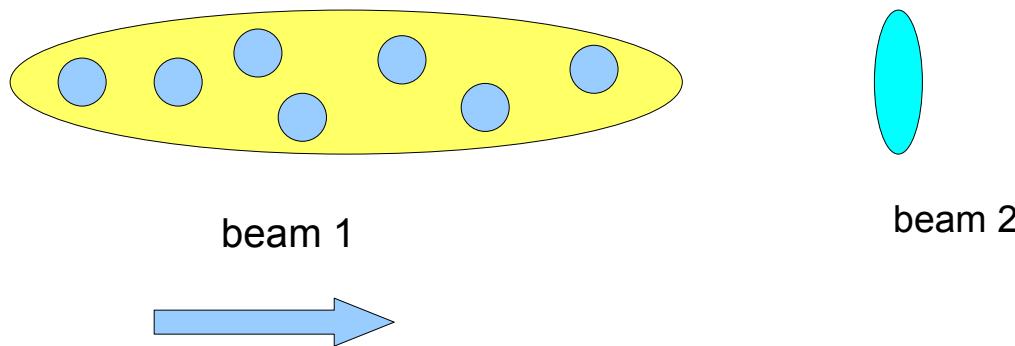
$\mu \equiv 2\pi\nu$ (one turn phase advance)

Eigenvalues of one-turn map give mode tunes (frequencies)

$$M_{\text{one-turn}} \begin{pmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{pmatrix} = \lambda \begin{pmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{pmatrix} \quad \lambda = e^{\pm 2\pi i v} \quad v = \text{tune} \quad (\text{bbccalc.m script})$$

Beam-beam simulations with PIC code

short bunches 2-D simulation



For ultra-relativistic beams, in beam 1's reference frame, the electric field from beam 2 is Lorentz-contracted into a pancake, so beam 1 receives a transverse impulse as it flies by.

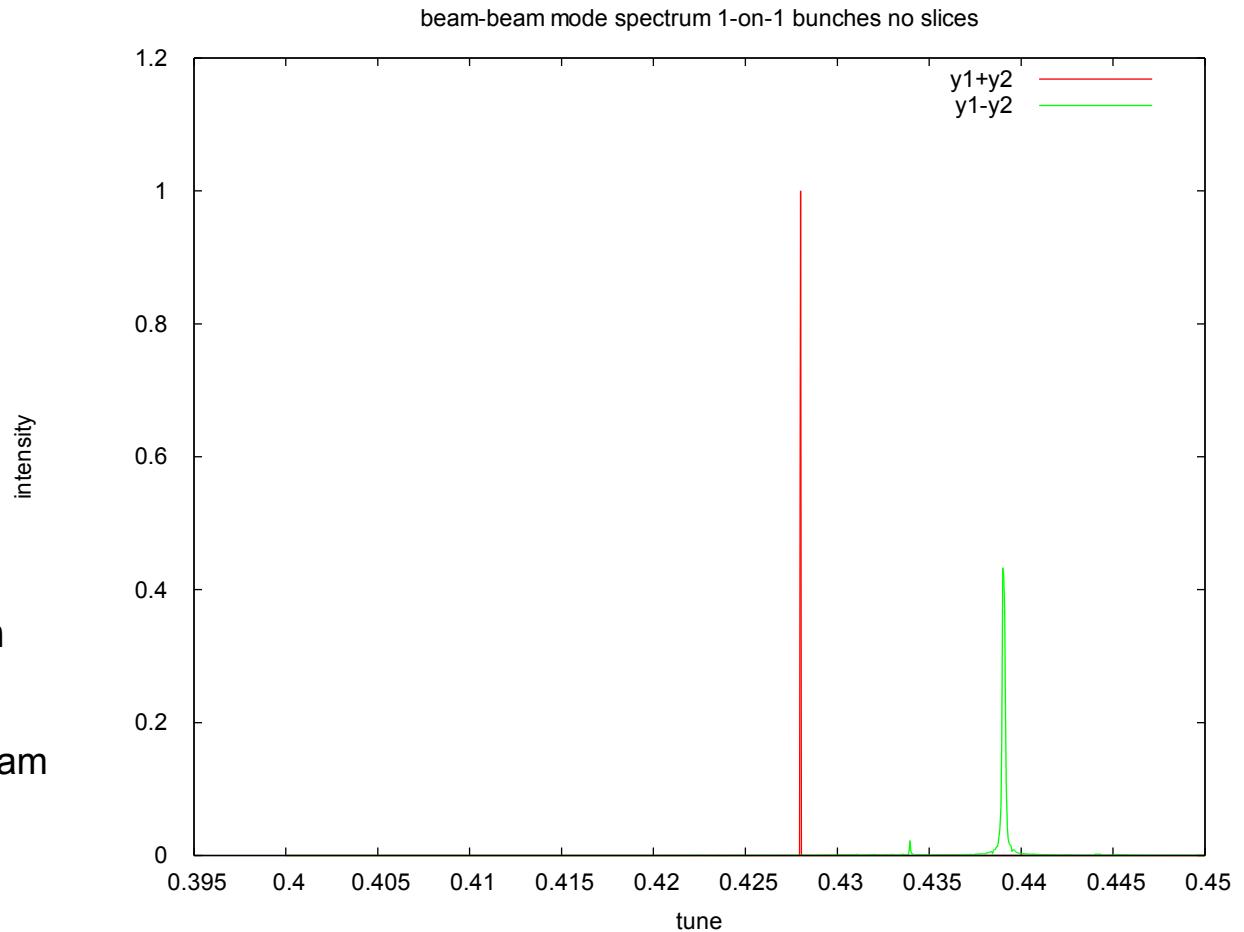
Solve 2-D Poisson equation for beam 2's electric field, apply force to beam 1's particles, and vice-versa.

Example run, BeamBeam3d code (E. Stern FNAL,J. Qiang, LBNL)

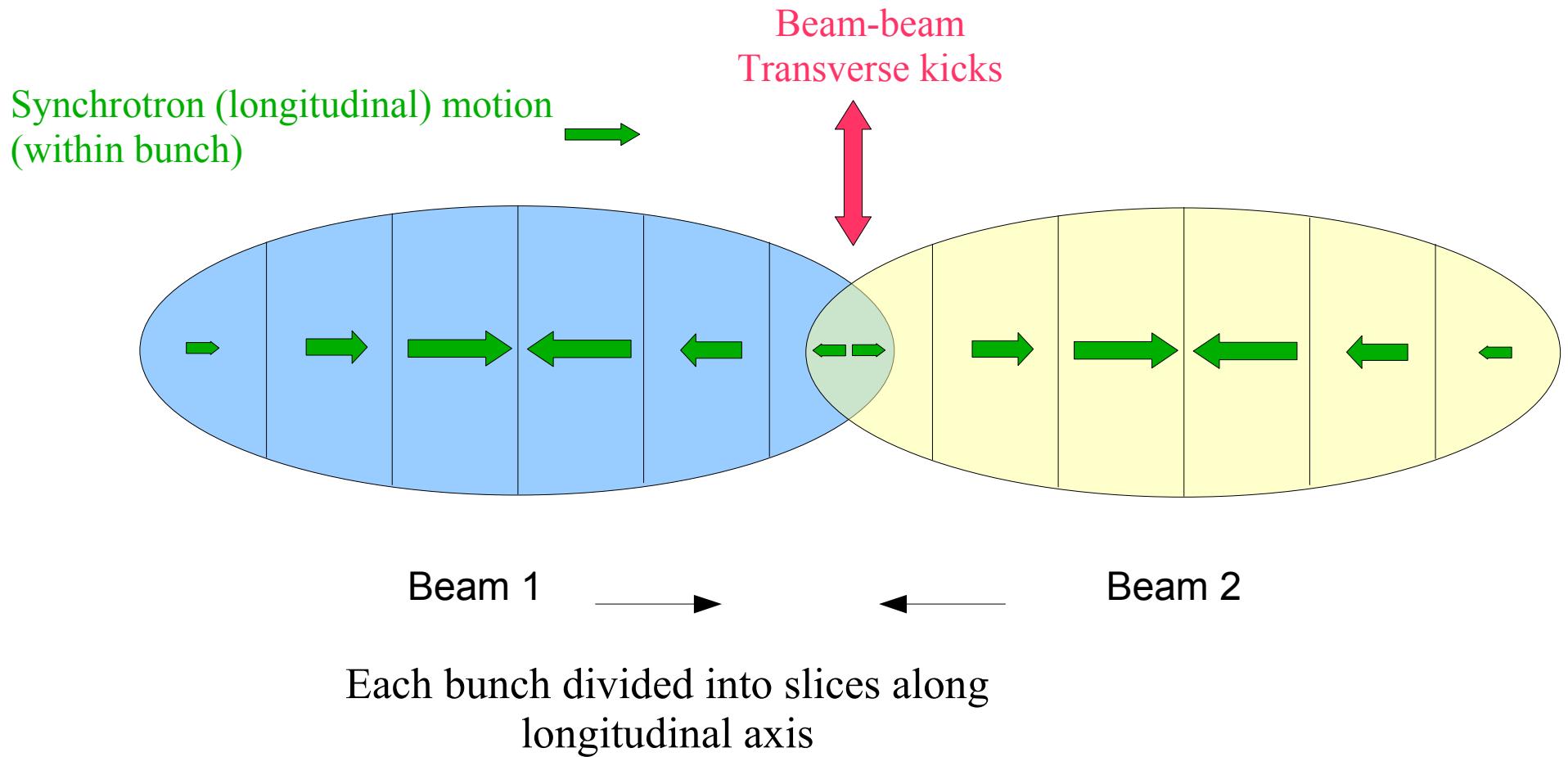
Intensity	$1.6 \cdot 10^{12} p/\text{bunch}$
Energy	150 GeV
Base tune	0.428
RMS Beam width	76 μm (x) 7.6 μm (y)
β (twiss)	0.28 m
ξ	0.011
σ mode peak	0.428
π mode peak	0.439

Matches results from `bbcalc.m` script.

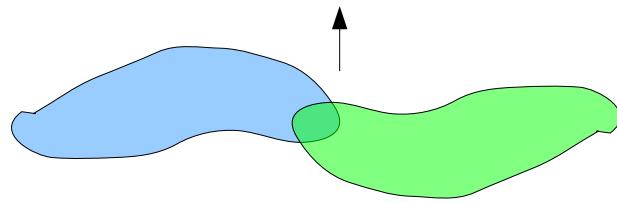
Actual dependence on beam-beam parameter depends on beam shape.



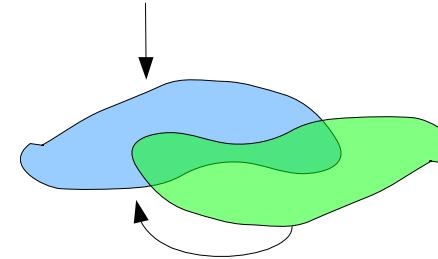
3D (almost) beam-beam calculation



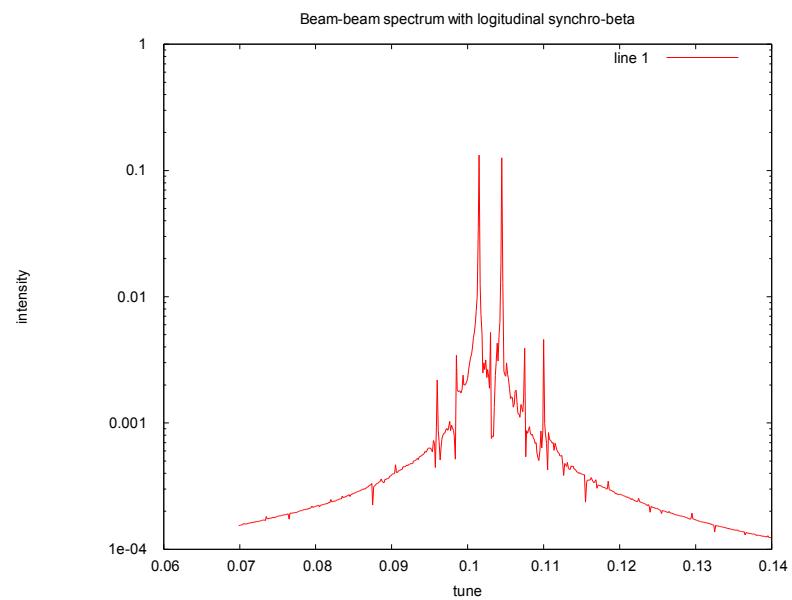
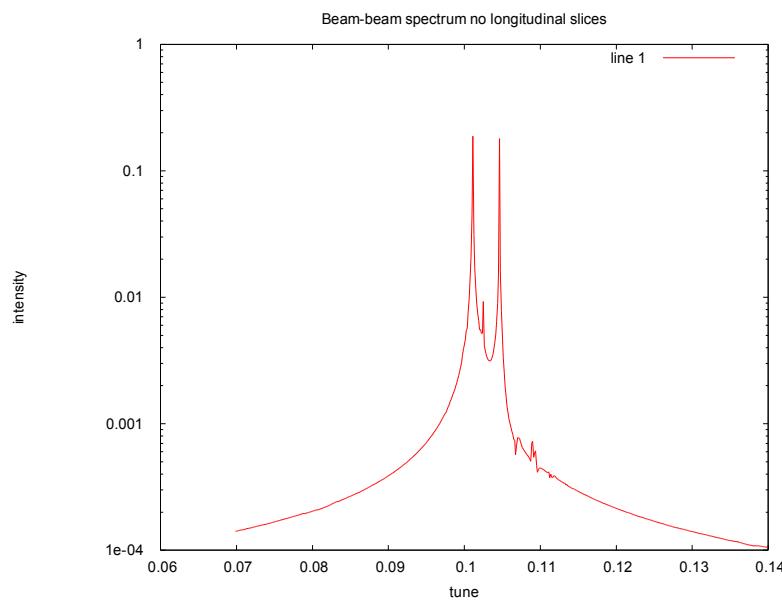
Synchro-betatron modes are coupled oscillations where the head of a beam bunch couples to the tail mediated by beam-beam interactions with bunches from another beam.



no longitudinal motion



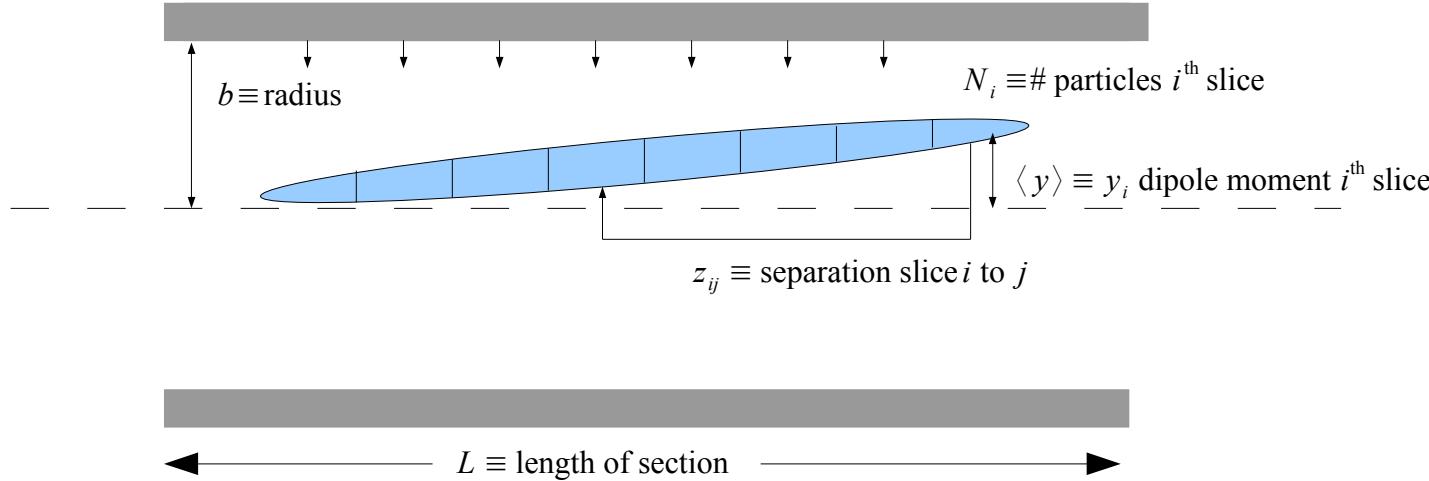
longitudinal motion with
synchro-betatron modes



the main peaks from beam-beam splitting are duplicated above and below the main peak offset by the synchrotron tune (.007)

Wakefields

Resistive wall beam pipe conductivity= σ



Off-axis charges induce wakefields behind leading edge of bunch that effects trailing edge.

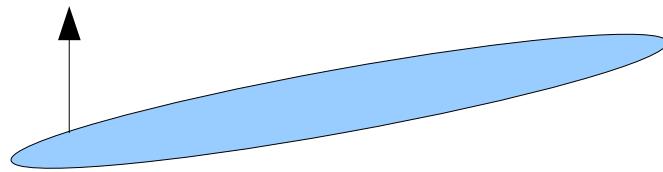
Dipole resistive wall wake function

$$W_1 = \frac{2}{\pi b^3} \sqrt{\frac{c}{\sigma}} \frac{L}{\sqrt{z_{ij}}}$$

Momentum kick on particle in trailing slice j from leading slice i

$$\Delta y_j' = N_i r_p \langle y_i \rangle W_1$$

Effects of Wakefields: Strong head-tail instability



Wake force on trailing particles is always in the same direction as the leading edge offset from the beam axis and proportional to the bunch charge..

When the wake force overcomes the restoring force from the lattice and the head-tail mixing from synchrotron motion, the beam becomes unstable.

$$\text{Stability requirement } Y = \frac{N r_p W_1 L}{8 \pi \gamma v_s v_\beta} < 2$$

